Math 60 **7.6 Simplifying Complex Rational Expressions**

Objectives:

1) Identify a complex fraction (or "fraction-within-fraction") and recognize that complex fractions are NEVER simplified final answers.
2) Simplifying Complex Rational Expressions by Dividing (Method I)
   a. Rewrite the expression as fraction $\div$ fraction.
   b. Rewrite as multiply by the reciprocal.
   c. Multiply the two rational expressions by factor and cancel (7.1)
3) Simplifying Complex Rational Expressions by Multiplying by the LCD (Method I)
   a. Identify the denominator(s) of numerator fractions and denominator(s) of denominator fractions.
   b. Find the LCD of all the denominators found.
   c. Multiply the complex fraction by $1 = \dfrac{LCD}{LCD} = \dfrac{\frac{LCD}{1}}{\frac{LCD}{1}}$

Examples and Practice:

Simplify completely, using both methods to solve each problem.

1) $\dfrac{\frac{4}{x+3}}{\frac{8}{x^2-9}}$

2) $\dfrac{\frac{1}{5}+\frac{1}{x}}{\frac{x+5}{2}}$

3) $\dfrac{\frac{2}{a^2}+\frac{3}{a}}{\frac{1}{a^3}-\frac{4}{a}}$

4) $\dfrac{\frac{1}{x+3}+1}{x-\frac{4}{x+3}}$

5) $\dfrac{\frac{2}{y}-\frac{8}{y^3}}{y^2+\frac{1}{y}}$

6) $\dfrac{\frac{-6}{y^2+5y+6}}{\frac{2}{y+3}-\frac{3}{y+2}}$

7) $\dfrac{2-\frac{3}{x}-\frac{2}{x^2}}{1-\frac{5}{x}+\frac{6}{x^2}}$

8) $\dfrac{\frac{5}{2b+3}+\frac{1}{2b-3}}{\frac{6b}{8b^2-18}}$

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① $\dfrac{\left(\dfrac{4}{x+3}\right)}{\left(\dfrac{8}{x^2-9}\right)} \longrightarrow \dfrac{4}{x+3} \div \dfrac{8}{x^2-9}$ Method 1 Divide fractions

$= \dfrac{4}{x+3} \cdot \dfrac{x^2-9}{8}$ mult by reciprocal

$= \dfrac{4}{\cancel{(x+3)}} \cdot \dfrac{\cancel{(x+3)}(x-3)}{8}$ factor and cancel

$= \dfrac{4}{8}(x-3)$ reduce coefficients.

$= \boxed{\dfrac{1}{2}(x-3)}$ or $\boxed{\dfrac{x-3}{2}}$

Method 2: Multiply by $1 = \dfrac{LCD}{LCD} = \dfrac{\left(\dfrac{LCD}{1}\right)}{\left(\dfrac{LCD}{1}\right)}$

Factor everything. and find LCD

$\dfrac{\dfrac{4}{(x+3)}}{\dfrac{8}{(x+3)(x-3)}}$ ← denominator $(x+3)$ ← denominator $(x+3)(x-3)$ } LCD= $(x+3)(x-3)$

$= \dfrac{\dfrac{4}{\cancel{(x+3)}}}{\dfrac{8}{\cancel{(x+3)}\cancel{(x-3)}}} \cdot \dfrac{\dfrac{\cancel{(x+3)}(x-3)}{1}}{\dfrac{\cancel{(x+3)}\cancel{(x-3)}}{1}}$ mult + cancel

$= \dfrac{\dfrac{4(x-3)}{1}}{\dfrac{8}{1}}$ notice that 1s can be removed $\dfrac{4(x-3)}{1} = 4(x-3)$

$= \dfrac{4(x-3)}{8} = \boxed{\dfrac{x-3}{2}}$ and $\dfrac{8}{1} = 8$

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(2) $$\frac{\left(\frac{1}{5}+\frac{1}{x}\right)}{\left(\frac{x+5}{2}\right)}$$ ←

Method 1:

Important! This fraction bar is a sneaky grouping symbol — you must work out $\left(\frac{1}{5}+\frac{1}{x}\right)$ before you divide — add before divide doesn't happen in order of operations unless there are parentheses

$$= \left(\frac{1}{5}+\frac{1}{x}\right) \div \left(\frac{x+5}{2}\right)$$ add first

$$= \left(\frac{1}{5}\cdot\frac{x}{x} + \frac{1}{x}\cdot\frac{5}{5}\right) \div \frac{x+5}{2}$$ find LCD

$$= \frac{x+5}{5x} \div \frac{x+5}{2}$$ add numerators

$$= \frac{\cancel{(x+5)}}{5x} \cdot \frac{2}{\cancel{(x+5)}}$$ multiply by reciprocal and cancel

$$= \boxed{\frac{2}{5x}}$$

Method 2:

$$\frac{\frac{1}{5}+\frac{1}{x}}{\frac{x+5}{2}}$$

← denominators 5, x
← denominator 2

$\Big\}$ LCD = $10x$

$$= \frac{\frac{1}{\cancel{5}}\cdot\frac{\cancel{10}^{2}x}{1} + \frac{1}{\cancel{x}}\cdot\frac{10\cancel{x}}{1}}{\frac{x+5}{\cancel{2}}\cdot\frac{\cancel{10}^{5}x}{1}}$$

← distribute $\frac{10x}{1}$ to both
← mult by $\frac{10x}{1}$

$$= \frac{2x+10}{(x+5)5x}$$ cancel $\frac{10}{5}$, $\frac{x}{x}$, $\frac{10}{2}$

$$= \frac{2\cancel{(x+5)}}{5x\cancel{(x+5)}} = \boxed{\frac{2}{5x}}$$ factor + cancel $2x+10$

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(3) $$\frac{\left(\frac{2}{a^2}+\frac{3}{a}\right)}{\left(\frac{1}{a^3}-\frac{4}{a}\right)}$$

<u>Method 1</u>: Sneaky grouping symbols!

$$= \underbrace{\left(\frac{2}{a^2}+\frac{3}{a}\right)}_{\text{add first}} \div \underbrace{\left(\frac{1}{a^3}-\frac{4}{a}\right)}_{\text{subtract 2nd}}$$

last

ORDER OF OPERATIONS

$$= \left(\frac{2}{a^2}+\frac{3a}{a^2}\right) \div \left(\frac{1}{a^3}-\frac{4a^2}{a^3}\right)$$

find LCD for + and find LCD for −

$$= \frac{2+3a}{a^2} \div \frac{1-4a^2}{a^3}$$

add numerators

multiply by reciprocal

$$= \frac{2+3a}{\cancel{a^2}} \cdot \frac{\cancel{a^3}^{\,a}}{1-4a^2}$$

$$\frac{a^3}{a^2} = a$$

$$= \frac{(3a+2)}{1} \cdot \frac{a}{-(2a-1)(2a+1)}$$

factor $1-4a^2 = -4a^2+1$
$= -(4a^2-1)$
$= -(2a-1)(2a+1)$

$$= \boxed{\frac{-a(3a+2)}{(2a-1)(2a+1)}}$$

best answer

<u>but</u> since nothing cancels, it's also ok to write

$$\boxed{\frac{a(3a+2)}{(1-2a)(1+2a)}}$$

<u>Method 2</u>:

$$\frac{\left(\frac{a^3}{1}\right)\frac{2}{a^2}+\frac{3}{a}\left(\frac{a^3}{1}\right)}{\left(\frac{a^3}{1}\right)\frac{1}{a^3}-\frac{4}{a}\left(\frac{a^3}{1}\right)}$$

denominators a^2, a

denominators a^3, a

LCD $= a^3$

$$= \frac{2a+3a^2}{1-4a^2} = \boxed{\frac{a(2+3a)}{(1-2a)(1+2a)}}$$

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Method 1: Must use parentheses.

4) $$\frac{\left(\frac{1}{x+3} + 1\right)}{\left(x - \frac{4}{x+3}\right)}$$

ORDER OF OPERATIONS

$$= \underbrace{\left(\frac{1}{x+3} + 1\right)}_{\text{add 1st}} \div \underbrace{\left(x - \frac{4}{x+3}\right)}_{\text{subtract 2nd}}$$

divide last

$$= \left(\frac{1}{x+3} + \frac{x+3}{x+3}\right) \div \left(\frac{x(x+3)}{x+3} - \frac{4}{x+3}\right)$$

find LCD + rewrite

$$= \left(\frac{x+4}{x+3}\right) \div \left(\frac{x^2+3x-4}{x+3}\right)$$

add numerators

$$= \frac{\cancel{(x+4)}}{\cancel{(x+3)}} \cdot \frac{\cancel{(x+3)}}{\cancel{(x+4)}(x-1)}$$

mult by reciprocal

factor x^2+3x-4 $= (x+4)(x-1)$

-4, 4, -1, 3

$$= \boxed{\frac{1}{(x-1)}}$$

Notice: all factors in numerator cancel out, so must write 1

Method 2:

$$\frac{\frac{\cancel{(x+3)}}{1} \cdot \frac{1}{\cancel{(x+3)}} + 1 \cdot \frac{(x+3)}{1}}{\frac{(x+3)}{1} \cdot x - \frac{4}{\cancel{(x+3)}} \cdot \frac{\cancel{(x+3)}}{1}}$$

denom $x+3$

denom $x+3$

LCD $= x+3$

mult $\frac{x+3}{1}$ to each fraction

$$= \frac{1 + (x+3)}{x(x+3) - 4}$$

cancel

combine numerator, dist & combine denominator

$$= \frac{\cancel{x} + 4}{x^2+3x-4}$$

factor x^2+3x-4

$(x+4)(x-1)$

-4, 4, -1, 3

$$= \frac{\cancel{(x+4)}}{\cancel{(x+4)}(x-1)} = \boxed{\frac{1}{x-1}}$$

cancel

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5 $\dfrac{\left(\frac{2}{y}-\frac{8}{y^3}\right)}{\left(y^2+\frac{1}{y}\right)}$

<u>Method 1:</u>

$= \left(\frac{2}{y}-\frac{8}{y^3}\right) \div \left(y^2+\frac{1}{y}\right)$ rewrite as $\div$

$= \left(\frac{2y^2-8}{y^3}\right) \div \left(\frac{y^3+1}{y}\right)$ subtract with LCD
add with LCD

$= \left(\frac{2y^2-8}{y^3}\right) \cdot \left(\frac{y}{y^3+1}\right)$ multiply by reciprocal

$= \frac{2(y-2)(y+2)}{y^3} \cdot \frac{y}{(y+1)(y^2-y+1)}$

factor $2y^2-8$
$= 2(y^2-4)$ GCF
$= 2(y-2)(y+2)$ diff of sq.

factor y^3+1
sum of cubes
$(\underset{S}{y+1})(y^2 \underset{O}{-} y \underset{AP}{+} 1)$

$= \boxed{\frac{2(y-2)(y+2)}{y^2(y+1)(y^2-y+1)}}$

cancel $\frac{y}{y^3} = \frac{1}{y^2}$

<u>Method 2:</u>

$$\frac{\frac{y^3}{1}\cdot\frac{2}{y} - \frac{8}{y^3}\cdot\frac{y^3}{1}}{\frac{y^3}{1}\cdot\frac{y^2}{1} + \frac{1}{y}\cdot\frac{y^3}{1}}$$

← denominators y + y^3
← denominators 1 & y

LCD = y^3

mult all by $\frac{LCD}{1}$

$= \frac{2y^2-8}{y^5+y^2}$

factor $2y^2-8 = 2(y^2-4)$ GCF
$= 2(y-2)(y+2)$ diff of sq

$= \boxed{\frac{2(y-2)(y+2)}{y^2(y+1)(y^2-y+1)}}$

factor $y^5+y^2 = y^2(y^3+1)$ GCF
$= y^2(y+1)(y^2-y+1)$ sum cubes

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⑥ $$\frac{\left(\dfrac{-6}{y^2+5y+6}\right)}{\left(\dfrac{2}{y+3}-\dfrac{3}{y+2}\right)}$$

Method 1:

Write with ÷ and parentheses

$$= \left(\frac{-6}{y^2+5y+6}\right) \div \left(\frac{2}{y+3} - \frac{3}{y+2}\right)$$

order of operations: subtract before ÷ because of ( ).

$$= \underbrace{\frac{-6}{y^2+5y+6}}_{\text{unchanged}} \div \left(\frac{2}{(y+3)}\cdot\frac{(y+2)}{(y+2)} - \frac{3}{(y+2)}\cdot\frac{(y+3)}{(y+3)}\right)$$

find LCD and rewrite fractions

$$= \frac{-6}{y^2+5y+6} \div \left(\frac{2y+2-(3y+9)}{(y+2)(y+3)}\right)$$

dist $2(y+2)$
dist $3(y+3)$
subtract numerators, using ( ) to dist neg next

$$= \frac{-6}{y^2+5y+6} \div \left(\frac{2y+2-3y-9}{(y+2)(y+3)}\right)$$

dist neg

$$= \frac{-6}{y^2+5y+6} \div \left(\frac{-y-7}{(y+2)(y+3)}\right)$$

combine like terms

$$= \frac{-6}{\cancel{(y+3)}\cancel{(y+2)}} \cdot \frac{\cancel{(y+2)}\cancel{(y+3)}}{-(y+7)}$$

factor y^2+5y+6 $\begin{array}{ccc} & 6 & \\ 2 & \times & 3 \\ & 5 & \end{array}$
$= (y+2)(y+3)$
mult by reciprocal
factor $-y-7 = -(y+7)$

$$= \boxed{\frac{6}{y+7}}$$

cancel $\frac{(-)}{(-)}$, $\frac{(y+2)}{(y+2)}$, $\frac{(y+3)}{(y+3)}$

Method 2:

$$\frac{\dfrac{-6}{y^2+5y+6}}{\dfrac{2}{y+3} - \dfrac{3}{y+2}}$$

factor y^2+5y+6 $\begin{array}{ccc} & 6 & \\ 2 & \times & 3 \\ & 5 & \end{array}$
$= (y+2)(y+3)$

find LCD $= (y+2)(y+3)$

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$$= \frac{\frac{-6}{\cancel{(y+2)}\cancel{(y+3)}} \cdot \frac{\cancel{(y+2)}\cancel{(y+3)}}{1}}{\frac{(y+2)\cancel{(y+3)}}{1} \cdot \frac{2}{\cancel{(y+3)}} - \frac{3}{\cancel{(y+2)}} \cdot \frac{\cancel{(y+2)}(y+3)}{1}}$$

mult. all by $\frac{\frac{LCD}{1}}{\frac{LCD}{1}}$

cancel common factors

$$= \frac{-6}{2(y+2) - 3(y+3)}$$

recopy remaining expression

$$= \frac{-6}{2y + 2 - 3y - 9}$$

dist $2(y+2)$ and $-3(y+3)$

$$= \frac{-6}{-y-7}$$

combine like terms

$$= \frac{-6}{-(y+7)}$$

factor -1 GCF
$-y-7 = -(y+7)$

$$= \boxed{\frac{6}{y+7}}$$

cancel $\frac{(-)}{(-)}$

(7) $$\frac{\left(2 - \frac{3}{x} - \frac{2}{x^2}\right)}{\left(1 - \frac{5}{x} + \frac{6}{x^2}\right)}$$

Method 1:
add ( ) around numerator and denominator

$$= \left(2 - \frac{3}{x} - \frac{2}{x^2}\right) \div \left(1 - \frac{5}{x} + \frac{6}{x^2}\right)$$

rewrite using ÷ symbol

$$= \left(\frac{2x^2}{x^2} - \frac{3x}{x^2} - \frac{2}{x^2}\right) \div \left(\frac{x^2}{x^2} - \frac{5x}{x^2} + \frac{6}{x^2}\right)$$

find LCD and rewrite.

$$= \frac{2x^2 - 3x - 2}{x^2} \div \frac{x^2 - 5x + 6}{x^2}$$

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$$= \frac{2x^2-3x-2}{x^2} \cdot \frac{x^2}{x^2-5x+6}$$

multiply by reciprocal
cancel x^2

factor $2x^2-3x-2$ -4, -4, $+1$, -3

$= 2x^2-4x+x-2$

$= 2x(x-2)+1(x-2)$

$= (x-2)(2x+1)$

factor x^2-5x+6 6, -2, -3, -5

$(x-2)(x-3)$

$$= \frac{(2x+1)\cancel{(x-2)}}{\cancel{(x-2)}(x-3)}$$

cancel $\frac{(x-2)}{(x-2)}$

$$= \boxed{\frac{2x+1}{x-3}}$$

Method 2

$$\frac{2-\frac{3}{x}-\frac{2}{x^2}}{1-\frac{5}{x}+\frac{6}{x^2}}$$

find LCD x^2
mult all six fractions by $\frac{x^2}{1}$

$$= \frac{2\cdot\frac{x^2}{1} - \frac{3}{x}\cdot\frac{x^2}{1} - \frac{2}{x^2}\cdot\frac{x^2}{1}}{1\cdot\frac{x^2}{1} - \frac{5}{x}\cdot\frac{x^2}{1} + \frac{6}{x^2}\cdot\frac{x^2}{1}}$$

cancel $\frac{x^2}{x} = x$
and $\frac{x^2}{x^2} = 1$

$$= \frac{2x^2-3x-2}{x^2-5x+6}$$

factor $2x^2-3x-2$
(see above)

factor x^2-5x+6
(see above)

$$= \frac{(2x+1)\cancel{(x-2)}}{\cancel{(x-2)}(x-3)}$$

$$= \boxed{\frac{2x+1}{x-3}}$$

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(8) $$\frac{\left(\frac{5}{2b+3}+\frac{1}{2b-3}\right)}{\left(\frac{6b}{8b^2-18}\right)}$$

Method 1:

Add ( ) around numerator and denominator

$$=\left(\frac{5}{2b+3}+\frac{1}{2b-3}\right)\div\left(\frac{6b}{8b^2-18}\right)$$ rewrite with ÷ symbol

$$=\left(\frac{5}{(2b+3)}\cdot\frac{(2b-3)}{(2b-3)}+\frac{1}{(2b-3)}\cdot\frac{(2b+3)}{(2b+3)}\right)\div\left(\frac{6b}{8b^2-18}\right)$$ find LCD and rewrite

$$=\frac{10b-15+2b+3}{(2b+3)(2b-3)}\div\frac{6b}{8b^2-18}$$ dist $5(2b-3)$ add numerators

$$=\frac{12b-12}{(2b+3)(2b-3)}\cdot\frac{8b^2-18}{6b}$$ Combine like terms mult by reciprocal.

$$=\frac{12(b-1)}{(2b\cancel{+3})(2b\cancel{-3})}\cdot\frac{2(2b\cancel{-3})(2b\cancel{+3})}{6b}$$

factor $8b^2-18$

$= 2(4b^2-9)$ GCF

$= 2(2b-3)(2b+3)$ diff of squares

cancel $\frac{2b-3}{2b-3}$; $\frac{2b+3}{2b+3}$

$\frac{12}{6}=2$

$$=\frac{2(b-1)\cdot 2}{b}$$

$$=\boxed{\frac{4(b-1)}{b}}$$

multiply coefficients

Method 2:

$$\frac{\frac{5}{2b+3}+\frac{1}{2b-3}}{\frac{6b}{8b^2-18}}$$

factor $8b^2-18$

$= 2(4b^2-9)$ GCF

$= 2(2b-3)(2b+3)$ diff of sq.

find LCD $= 2(b-3)(2b+3)$

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$$\frac{\frac{2\cancel{(2b+3)}(2b-3)}{1}\cdot\frac{5}{\cancel{(2b+3)}}+\frac{1}{\cancel{(2b-3)}}\cdot\frac{2(2b+3)\cancel{(2b-3)}}{1}}{\frac{6b}{\cancel{2}\cancel{(2b-3)}\cancel{(2b+3)}}\cdot\frac{\cancel{2}\cancel{(2b+3)}\cancel{(2b-3)}}{1}}$$

mult all by $\frac{\frac{LCD}{1}}{\frac{LCD}{1}}$

$$= \frac{2\cdot5\cdot(2b-3)+2(2b+3)}{6b}$$

copy remaining expression

$$= \frac{20b-30+4b+6}{6b}$$

dist $10(2b-3)$
$2(2b+3)$

$$= \frac{24b-24}{6b}$$

combine like terms

$$= \frac{24}{6}\frac{(b-1)}{6}$$

factor GCF

$$= \boxed{\frac{4(b-1)}{6}}$$

Note: b and 6 are often given in the same question to trick students with sloppy handwriting.
Idea: Use B instead of b.

Note: In Method #2, we might have reduced

$$\frac{6b}{2(2b-3)(2b+3)} = \frac{3b}{(2b-3)(2b+3)}$$

before finding and multiplying by LCD